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$$F(x) = F(x_0) + F'(x_0)(x-x_0) + \frac{F''(x_0)}{2}(x-x_0)^2$$

$$= F(0) + F'(0)x + \frac{F''(0)}{2}x^2 \qquad F(0) = \int_0^0 f(t)\,dt$$

$$= 0 + f(0)x + \frac{f'(0)}{2}x^2 \qquad = 0$$

$$= f(0)x + \frac{f'(0)}{2}x^2 \qquad F'(x) = f(x)$$

∴ Assumptions are that f is differentiable because f has to be continuous to be integrable and f has to be differentiable to have a derivative.